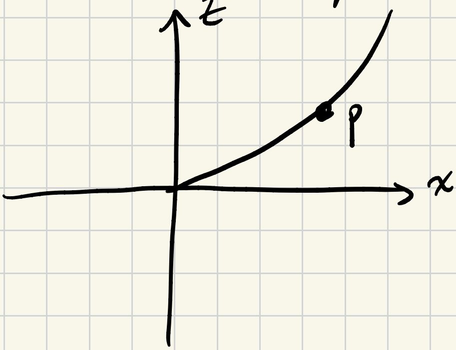
Week 9 Solutions

① a. $f_x = \dfrac{-y}{(x+y)^2}$ $\quad f_y = \dfrac{x}{(x+y)^2}$

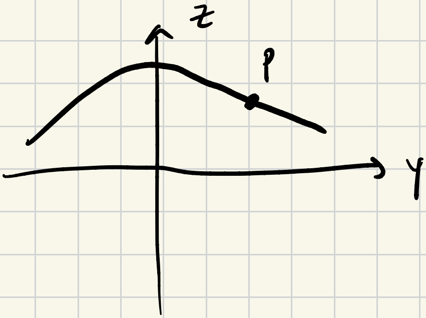
b. $f_x = \dfrac{2x+y}{x^2+xy}$ $\quad f_y = \dfrac{x}{x^2+xy}$

c. $f_x = \cos x \cos y$ $\quad f_y = -\sin x \sin y$

②



$\dfrac{\partial f}{\partial x} > 0$



$\dfrac{\partial f}{\partial y} < 0$

$$\begin{array}{r} 32 \\ \times\ 7 \\ \hline 14 \\ 210 \end{array} \qquad \begin{array}{c} 60-224 \\ 164 \end{array}$$

③ $f_x = 6xy + 12x^2y^2 - 7y^5$

$f_x(1,2) = 6\cdot 2 + 12\cdot 4 - 7\cdot 32 = 12 + 48 - 224 = -164$

④ $g_x = \dfrac{(x-y)y - xy}{(x-y)^2} = \dfrac{-y^2}{(x-y)^2}$

$g_{xy} = \dfrac{(x-y)^2\cdot(-2y) + 2y\cdot 2(x-y)(-1)}{(x-y)^4}$

$g_{xy}(1,0) = 0$

(5) $f_x = \dfrac{2x}{3y^2 + \ln(2+u^2)}$ $\qquad f_{xv} = 0$

(6) a. $\nabla f = \langle -2x \sin(x^2+y), -\sin(x^2+y) \rangle$

b. $\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

(7) a. $\nabla f = \langle 2x, 3y^2 \rangle$, $\nabla f(P) = \langle 2, 12 \rangle$

$u = v/\|v\| = \langle 4/5, 3/5 \rangle$

$D_u f = \nabla f(P) \cdot u = 2 \cdot 4/5 + 12 \cdot 3/5 = 8$

b. $\nabla f = \langle 2xy^3, 3x^2y^2 \rangle$, $\nabla f(P) = \langle 9, 3/4 \rangle$

$u = v/\|v\| = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$

$D_u f = \nabla f(P) \cdot u = \dfrac{1}{\sqrt{2}}(9 + 3/4) = \dfrac{39}{4\sqrt{2}}$

c. $\nabla g = \left\langle \ln(y+z), \dfrac{x}{y+z}, \dfrac{x}{y+z} \right\rangle$

$\nabla g(P) = \langle \ln(2)+1, 1/e, 1/e \rangle$

$u = v/\|v\| = \langle 2/\sqrt{6}, -1/\sqrt{6}, 1/\sqrt{6} \rangle$

$D_u g = \nabla g(P) \cdot u = \dfrac{2}{\sqrt{6}}(\ln(2)+1)$